Density matrix description of transport and gain in quantum cascade lasers in a magnetic field

Ivana Savić[1*], Nenad Vukmirović[1], Zoran Ikonić[1], Dragan Indjin[1], Paul Harrison[1], Robert W. Kelsall[1], Vitomir Milanović[2]

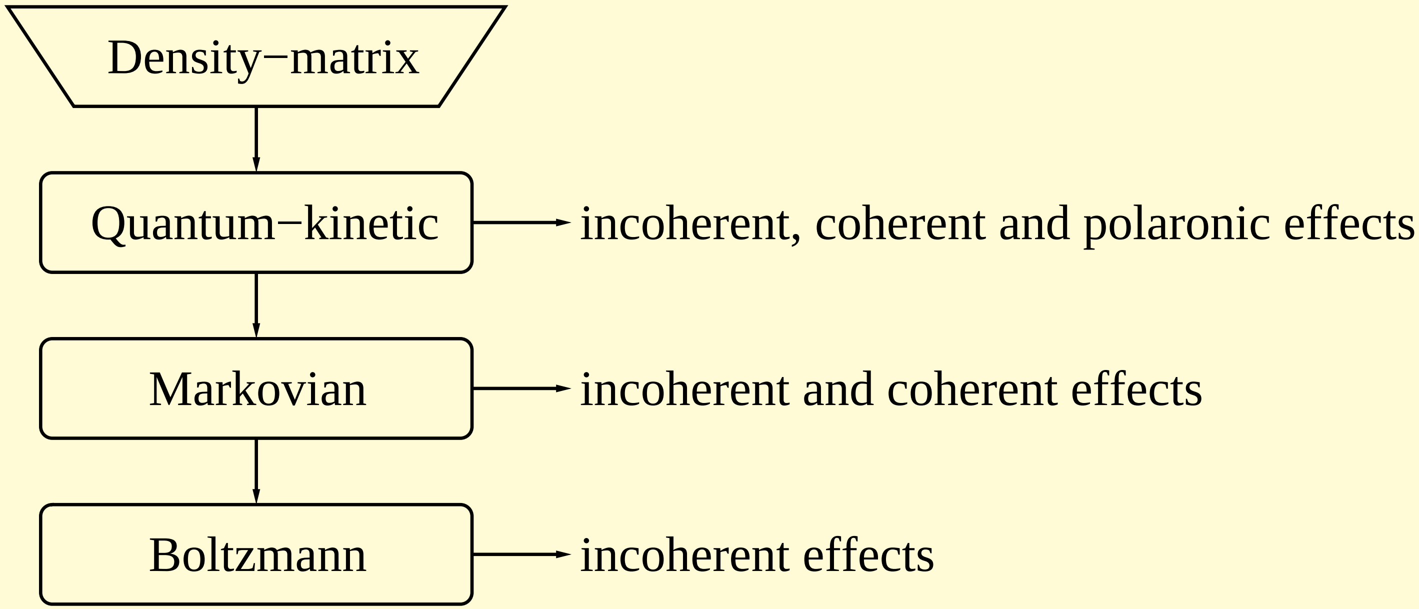
[1]Institute of Microwaves and Photonics,
School of Electronic and Electrical Engineering,
University of Leeds, U.K.

[2]Faculty of Electrical Engineering,
University of Belgrade, Serbia

[*] email: I.Savic@leeds.ac.uk

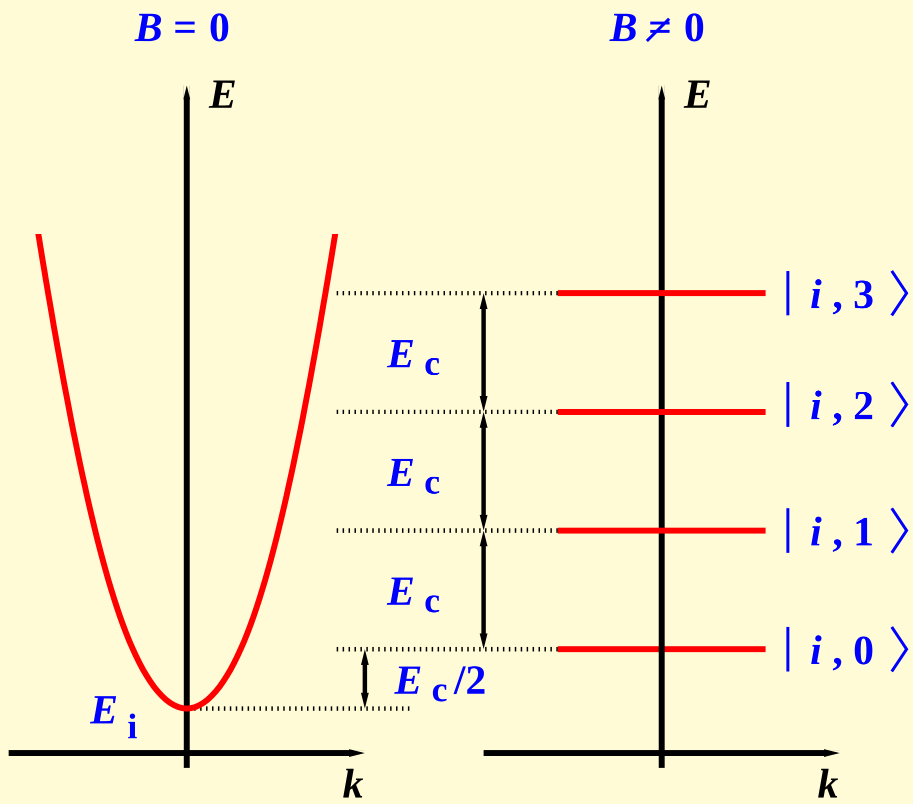
Objectives

- To investigate quantum-mechanical (coherent and polaronic) effects in QCLs in a magnetic field and their influence on:
  - Electron populations.
  - Output characteristics.
- To develop a quantum-mechanical theory of transport and optical properties of QCLs in a magnetic field.



QCLs in a magnetic field

- Discrete electronic structure (Landau levels).
- Scattering rates are significantly enhanced or reduced, depending on the Landau level (LL) configuration.
- Reduced scattering rates ⇒ improved performance:
  - Lower threshold current.
  - Larger population inversion and optical gain.

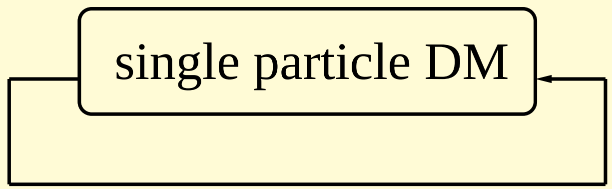


Electron transport in a cascade - density-matrix approach

- Single particle electron density matrices $f_{i_1 i_2,k} = \left\langle \hat{c}^{\dagger}_{i_1,k} \hat{c}_{i_2,k} \right\rangle$, $n_{i_1 i_2} = \sum_{k'} f_{i_1 i_2,k'}/L_x L_y$:
  - The diagonal elements - the occupation probabilities of LLs.
  - The non-diagonal elements - the quantum-mechanical coherence between LLs.
- The Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{H}_{el} + \hat{H}_{ep}$ (non-interacting electrons, the electron-light interaction, the electron-LO phonon interaction).
- Non-interacting electrons:

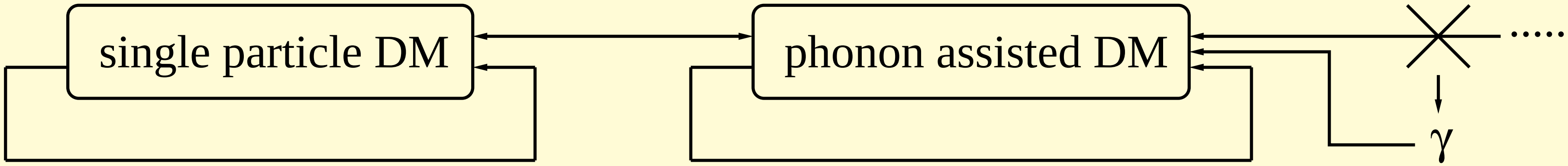
$$\frac{d}{dt} n_{i_1 i_2}\Big|_{\hat{H}_0} = \frac{1}{i\hbar}(E_{i_2} - E_{i_1}) n_{i_1 i_2}.$$

- Interaction of electrons with z-polarized light: Landau index conserved.



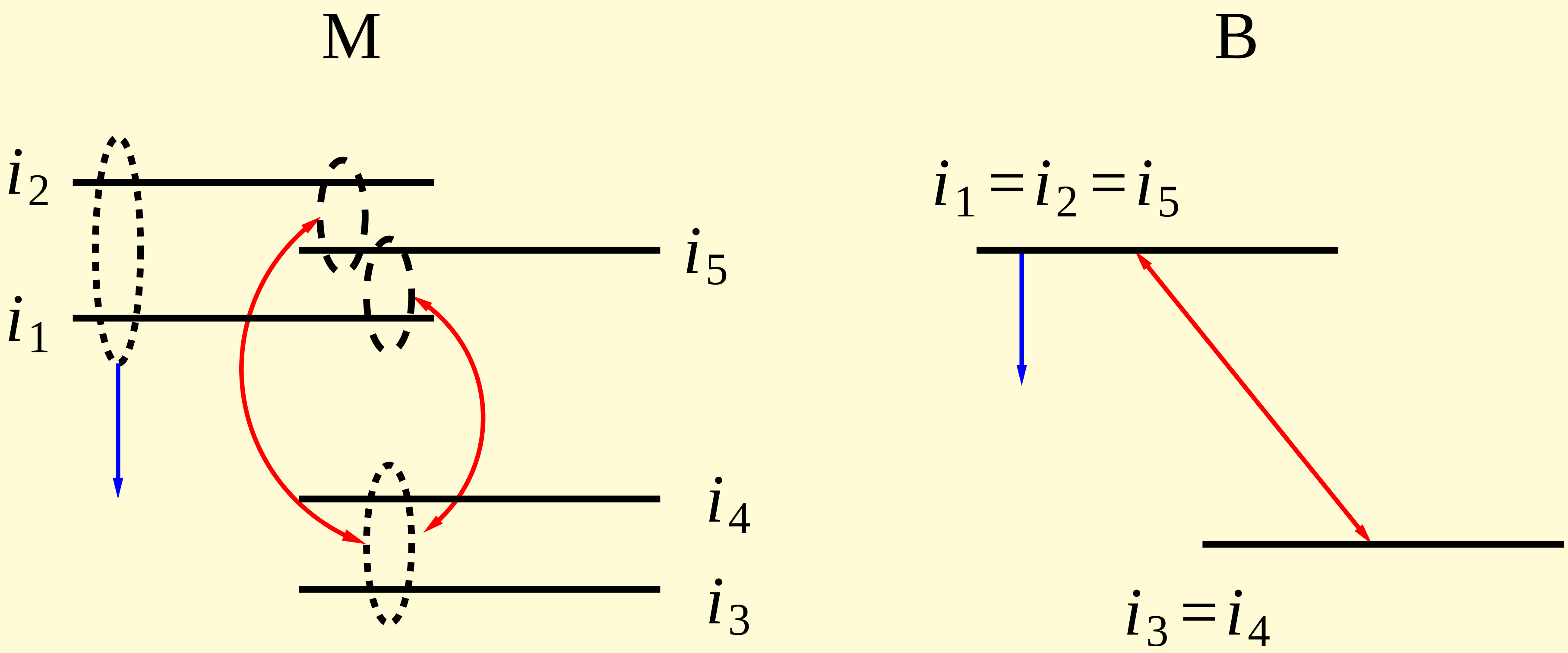
Interaction with LO phonons

- Quantum-kinetic (non-Markovian) description:
  - Phonon-assisted matrices $s^{i_1 i_2}_{k,\mathbf{q},k'} = \left\langle \hat{c}^{\dagger}_{i_1,k} \hat{b}_{\mathbf{q}} \hat{c}_{i_2,k'} \right\rangle$.



  - Broadening of LLs represented by a phenomenological damping constant γ.
- Markovian description:
  - Adiabatic elimination of phonon-assisted matrices $\Rightarrow$ the quantum-kinetic equations reduce to the Markovian equations.
  - Broadening of LLs - a Lorentzian with the FWHM of $2\hbar\gamma$.

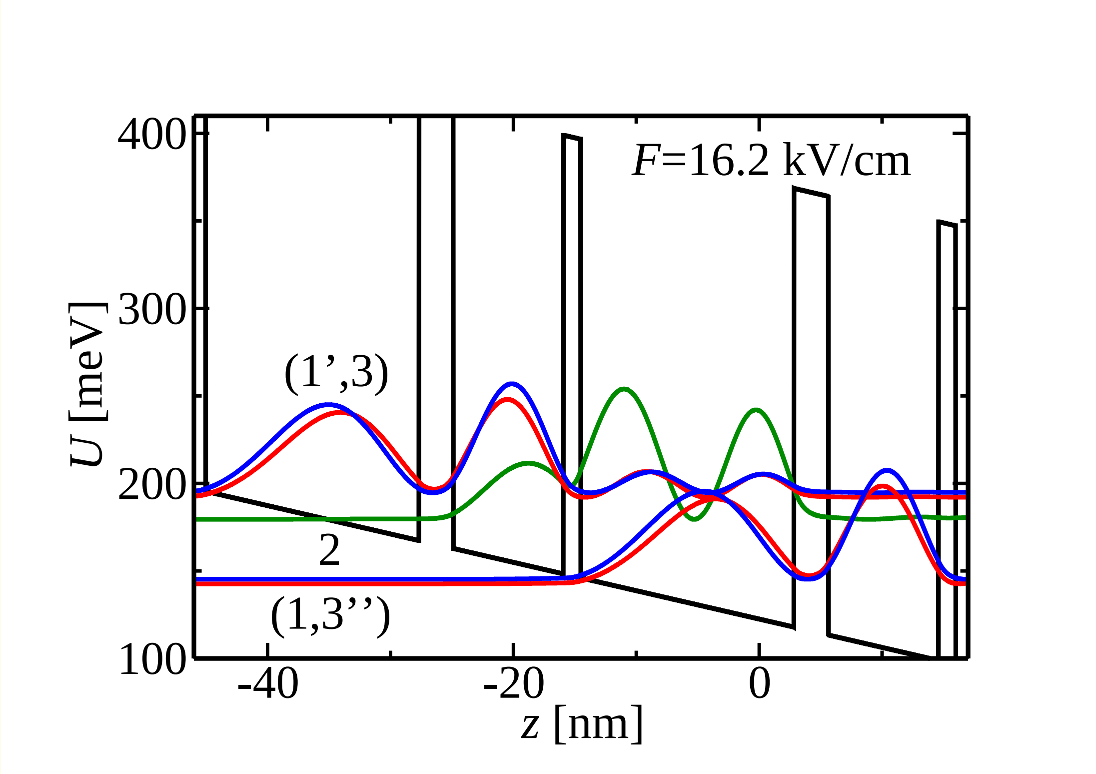
Interaction with LO phonons



- The semiclassical limit (non-diagonal matrix elements neglected) $\Rightarrow$ the Markovian equations reduce to the Boltzmann equations.
- The tight-binding description and the periodicity of the quantities involved were used in all three approaches.

QCL structure

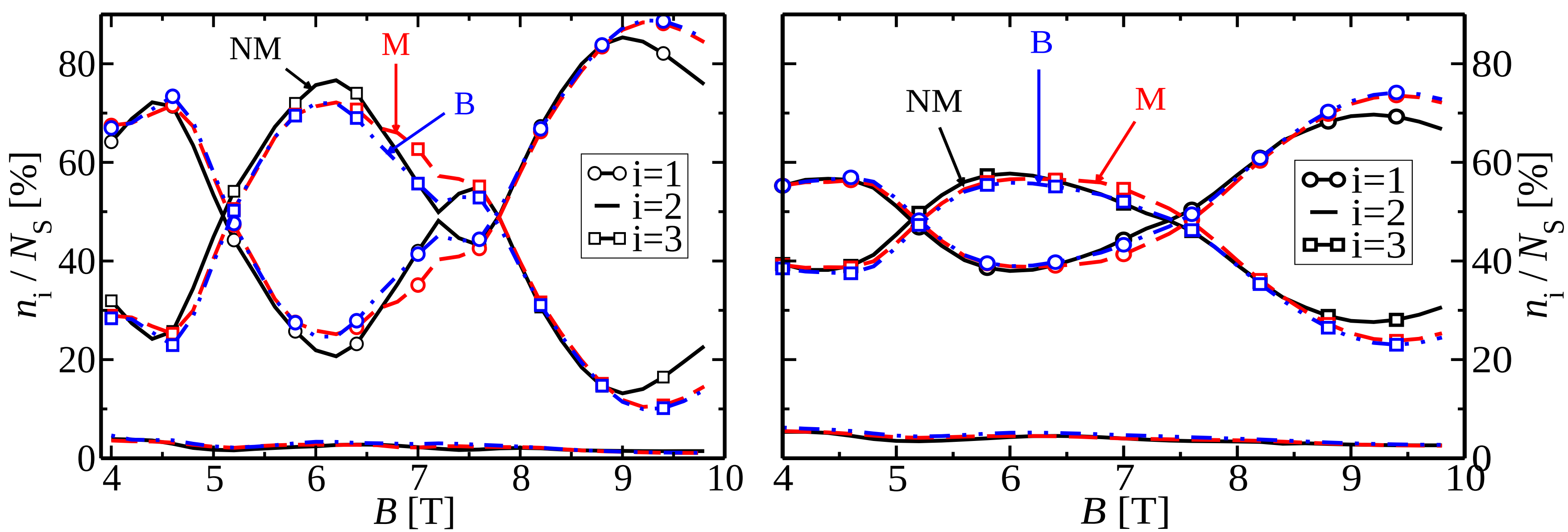
- Three-level scheme, LO phonon depopulation of the lower laser level, no injector.
- THz GaAs/$Al_{0.3}Ga_{0.7}As$, laser transition $\sim$ 15.2 meV.
- Dominant influence of the electron-LO phonon interaction on the populations.



The conduction band profile of the QCL.

Populations

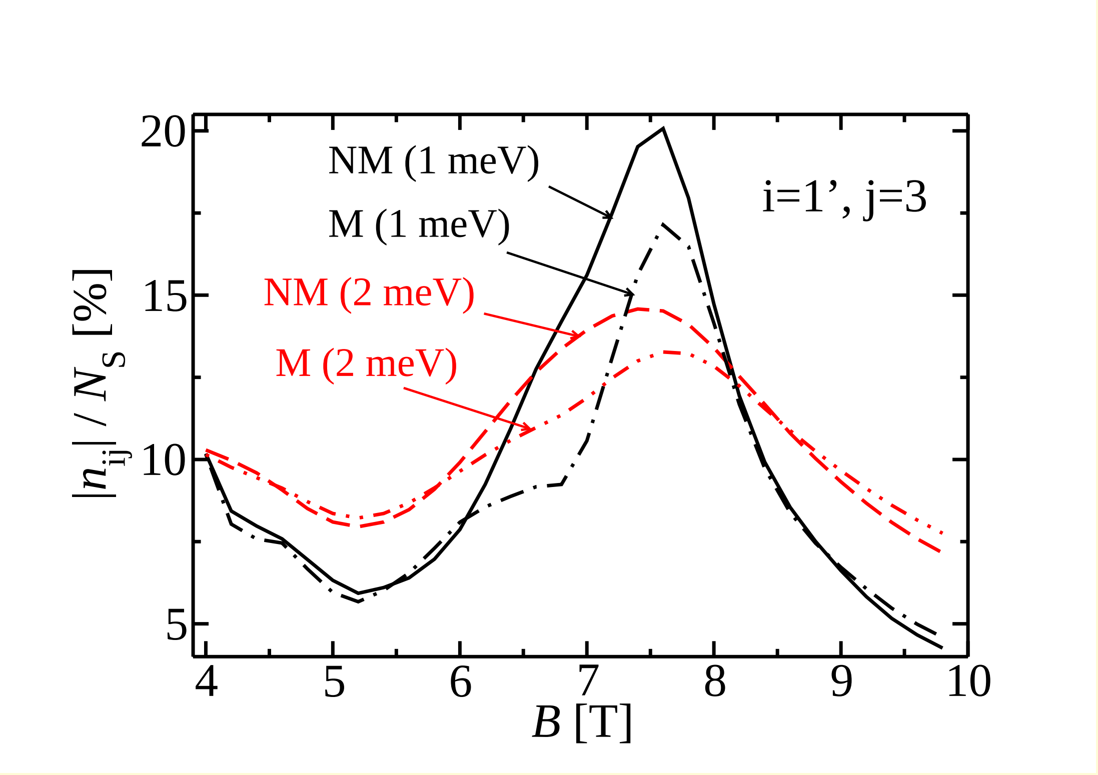
- Similar values of populations and their dependence on B from all three approaches.
- The differences due to coupling between populations and polarizations in the Markovian approach (among populations, polarizations and phonon-assisted matrices in the non-Markovian approach).



The electron population over QCL states (all Landau levels) vs magnetic field. Left: $\hbar\gamma = 1$ meV. Right: $\hbar\gamma = 2$ meV.

Polarizations

- Finite values of polarizations in the steady state.
- The largest polarization $\sim$ 10%.



The electron polarization between the ground state of the preceding period and the upper laser level (all Landau levels) vs magnetic field.

Nature of electron transport - coherent vs incoherent

- Quantum-mechanical (Markovian and non-Markovian) interpretation - coherent current.

$$J = -\frac{e}{d} \sum_{i_1,i_2=1}^{N} \left[v_{i_1 i_2} n_{i_2 i_1} + v_{i_1 (i_2+N)} n_{(i_2+N) i_1} + v_{(i_2+N) i_1} n_{i_1 (i_2+N)} \right],$$

$$v_{i_1 i_2} = \frac{i}{\hbar} \langle i_1 | [\hat{H}, \hat{z}] | i_2 \rangle = \frac{i}{\hbar} (E_{i_1} - E_{i_2}) z_{i_1 i_2} + \frac{1}{m^*} e A_R \delta_{i_1, i_2}.$$

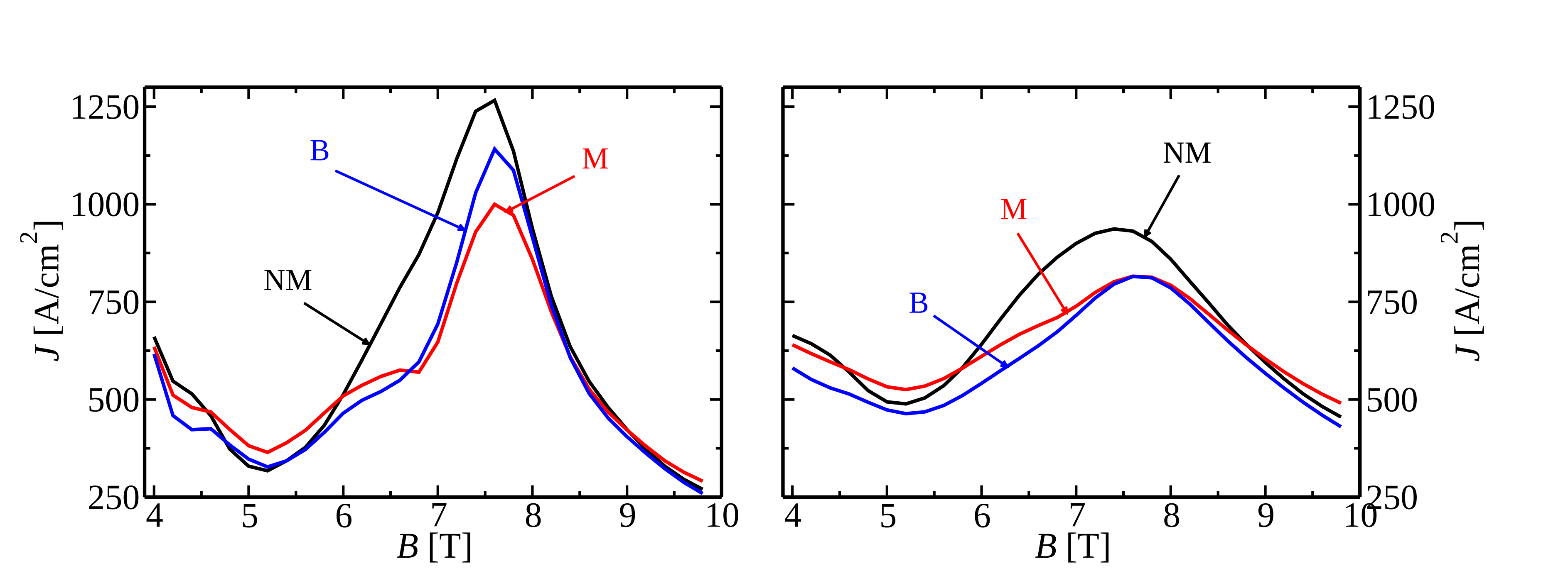
- Semiclassical interpretation - incoherent current.

$$J = \frac{e}{d} \left[\sum_{i=1}^{N} \sum_{\substack{f=1 \\ (i<f)}}^{2N} (z_f - z_i) \left[n_i W_{if} (1 - \alpha_B n_f) - n_f W_{fi} (1 - \alpha_B n_i) \right] \right].$$

Ref.: S. C. Lee, F. Banit, M. Woerner, and A. Wacker, Phys. Rev. B **73**, 245320 (2006).

Current

- Similar values of the current and its dependence on *B* in all three approaches.
- Relatively small coherences in the steady state.

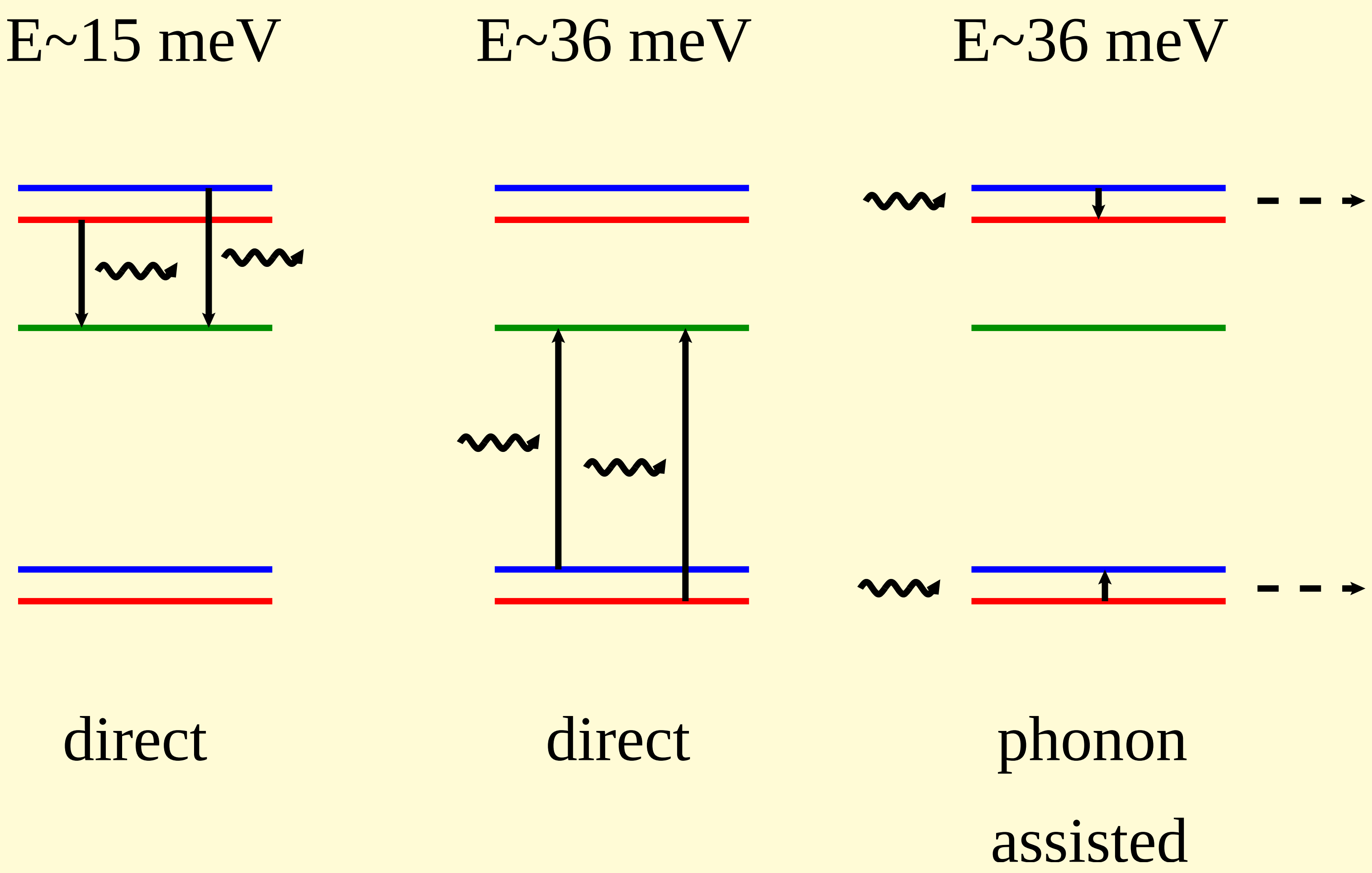


Current density vs magnetic field dependence. Left: $\hbar\gamma = 1$ meV. Right: $\hbar\gamma = 2$ meV.

Optical gain - general considerations

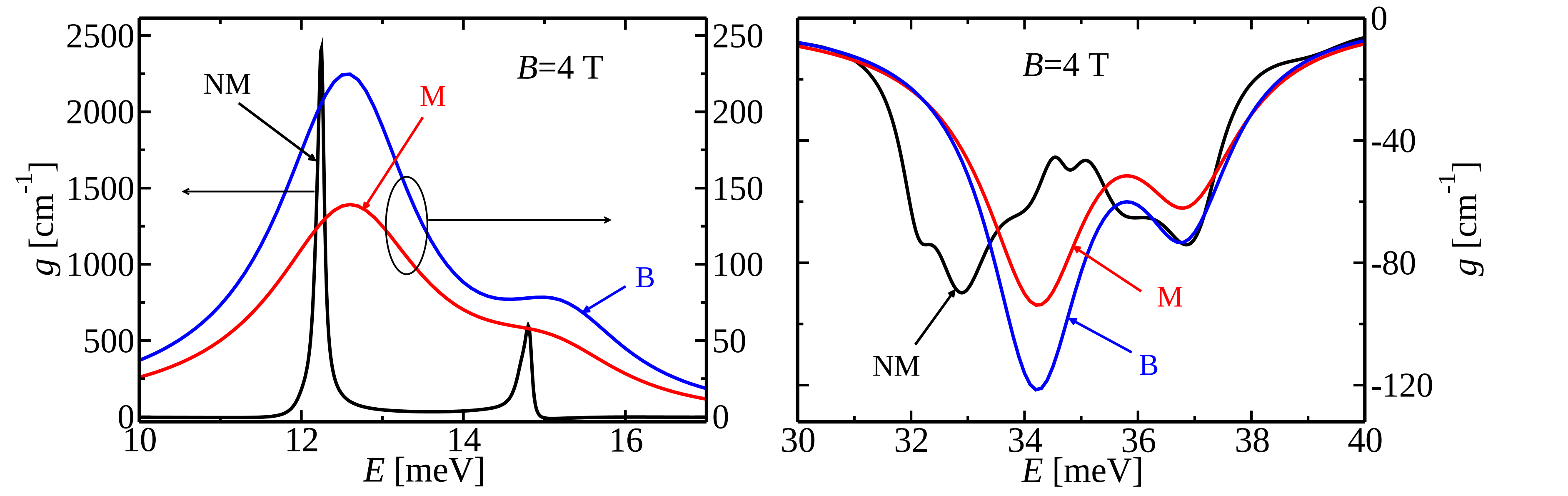
- Quantum-mechanical description: $g(\omega) \sim \mathrm{Im}[\chi(\omega)] \sim \mathrm{Im}[J(\omega)]$.
  - Non-Markovian approach:
    - Direct optical transitions - the gain linewidth determined by coupling to the LO phonon assisted transitions.
    - LO phonon assisted optical transitions $\sim \frac{1}{E_{i_2} \pm \hbar\omega_{\mathrm{LO}} - E_{i_1} \pm \hbar\omega - i\hbar\gamma}$ - the linewidth is of the order of $\sim 2\hbar\gamma$.
  - Markovian approach:
    - Direct optical transitions $\sim \frac{1}{E_{i_2} \pm \hbar\omega_{\mathrm{LO}} - E_{i_1} - i\hbar\gamma}$ - the linewidth determined by the scattering processes.
- Semiclassical description: Fermi's golden rule.
  - Direct optical transitions - the linewidth taken to be $2\hbar\gamma$.

Optical transitions



Optical gain - non-Markovian case

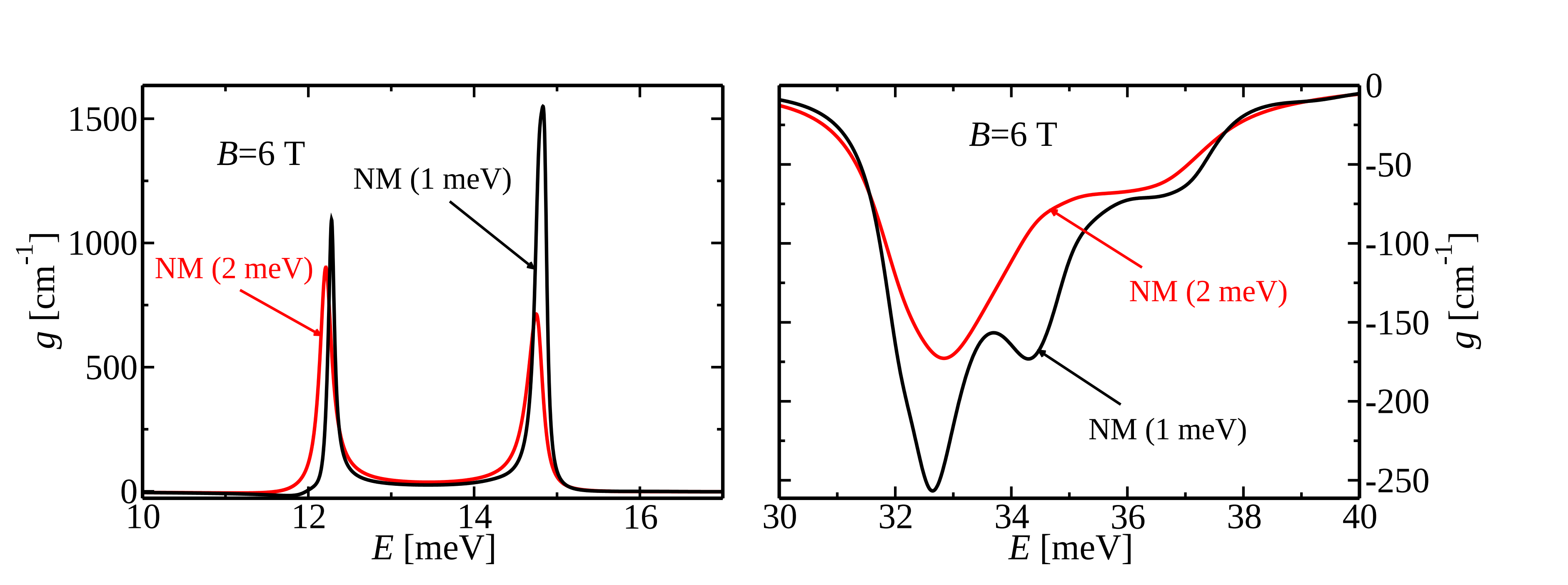
- The gain linewidth for the energies corresponding to the laser transitions is considerably smaller than for the energies around one LO phonon energy.
- Signatures of the polaron shift.



Optical gain vs energy for a magnetic field of 4 T and $\hbar\gamma = 1$ meV. Left: The energy range is in the vicinity of the optical transition energies. Right: The energy range is in the vicinity of one longitudinal optical phonon energy.

Optical gain - non-Markovian case

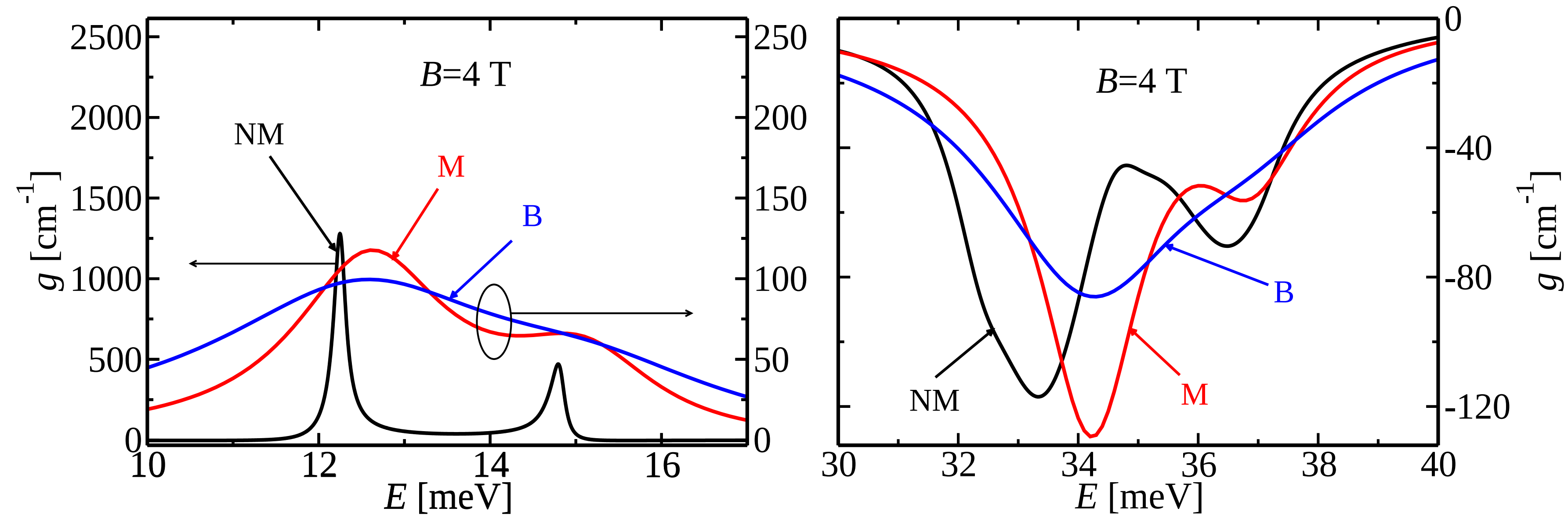
- Additional peaks for energies close to one LO phonon energy.
- A non-trivial interplay between the resonant LO phonon assisted transitions and resonant direct optical transitions for energies close to LO phonon energy.



Optical gain vs energy for a magnetic field of 6 T. Left: The energy range is in the vicinity of the optical transition energies. Right: The energy range is in the vicinity of one longitudinal optical phonon energy.

Optical gain - Markovian case

- Resonant scattering terms present throughout the energy range of interest.
- Large linewidth for the energies corresponding to the laser transitions and the energies around one LO phonon energy.



Optical gain vs energy for a magnetic field of 4 T and $\hbar\gamma = 2$ meV. Left: The energy range is in the vicinity of the optical transition energies. Right: The energy range is in the vicinity of one longitudinal optical phonon energy.

Summary

- Quantum-mechanical theory of gain and electron transport in QCLs in a magnetic field based on the density-matrix formalism:
  - Non-Markovian.
  - Markovian.
  - Boltzmann.
- Similar populations.
- Finite, but relatively small coherences.
- Comparable values of the current densities, despite different interpretations of the origin of the transport processes.
- Narrow linewidths for laser transitions and evidence of polaron formation in the non-Markovian treatment, in contrast to the Markovian and Boltzmann predictions.

Ref.: I. Savić, N. Vukmirović, Z. Ikonić, D. Indjin, R. W. Kelsall, P. Harrison, and V. Milanović, cond-mat/0702508, accepted for publication in Phys. Rev. B.